MATHEMATICAL MODELING OF AN AUTONOMOUS UNINHABITED UNDERWATER VEHICLE DYNAMICS WITH PROPELLER IN THE ROTARY NOZZLE

Abstract. In this paper, the research of hydrodynamic characteristics of a rotary screw column in the propulsion and steering complex (PSC) of an autonomous uninhabited underwater vehicle (AUV) is being carried out. These characteristics include: the change of the propeller thrust force and the force on the propeller shaft, during the change in the incident flow angle. To obtain the required characteristics, the method of mathematical modeling of the AUV plane motion in the Simulink system was applied. The improved mathematical model of the vehicle rectilinear motion was performed by supplementing its equations, which allows to simulate the operation of the PSC, “propeller–rotary nozzle”, in the downwash water flow. The propeller operation design in the guide nozzle in a direct and downwash water flow at the underwater vehicle different speeds is carried out. Vehicle speed and shaft speed graphs were obtained. The dependences of the change in propeller thrust force on incident flow angle are plotted for various underwater vehicle traverse speeds. The possibility of accounting for the significant nonlinear dependence of the propeller thrust force and the propeller shaft force on the incident flow angle is the scientific novelty. Practical significance of the work lies on the possibility of improving the AUV control quality by compensating for the changes in the propeller thrust force and force on the propeller shaft. It will be particularly relevant to use the obtained characteristics in the development of control systems for AUV trajectory motion.

Keywords: mathematical modeling; autonomous uninhabited underwater vehicle; propulsion and steering complex; rotary screw column.
Аннотация. Исследованы гидродинамические характеристики поворотной винтовой колонки в составе дви
gательно-рулевого комплекса (ДРК) автономного необитаемого подводного аппарата (АНПА). К таким ха
рактеристикам относятся: изменение упора гребного винта и усилия на гребном валу при изменении угla
набегающего потока. Для получения необходимых характеристик применена методика математического
моделирования плоского движения АНПА в системе Simulink. Усовершенствована математическая модель
прямолинейного движения аппарата дополнением ее уравнениями, позволяющими имитировать работу ДРК
«гребной винт–поворотная насадка» в скосшенном потоке воды. Проведено моделирование работы гребного
винта в направляющей насадке в прямом и скосшенном потоках воды при различной скорости движения под-
водного аппарата. Получено график скорости движения аппарата и график частоты вращения вала. Построены
зависимости изменения упора гребного винта от угла набегающего потока для различных скоростей движения
подводного аппарата. Построены зависимости изменения момента на валу гребного винта от угла набегающе-
го потока для различных скоростей движения подводного аппарата. Научной новизной является возможность
учета значительной нелинейности зависимости упора гребного винта и усилия на гребном валу от угла на-
bегающего потока. Практическая значимость работы заключается в возможности улучшения качества управ-
ления АНПА за счет компенсации изменения силы упора гребного винта и усилия на гребном валу. Особенно
актуально будет использование полученных характеристик при разработке систем управления траекторным
движением автономными аппаратами.

Ключевые слова: математическое моделирование; автономный необитаемый подводный аппарат; движи-
gательно-рулевой комплекс; поворотная винтовая колонка.

References
Problem statement. Nowadays, scientific research, emergency rescue, operational, industrial and other vehicles are created or developed, which differ considerably in appearance, overall dimensions and purpose, as well as in the principle of motion and control stabilization.

However, all varieties of AUV combine the general property of having the ability to move freely under water in one way or another distant (remote) from the surface. The forces acting on the AUV during motion determine its dynamics and significantly affect the controllability of the vehicle. Only having the full information about all the forces affecting the vehicle, as well as means of controlling them, can one determine the conditions under which all the dynamic conditions of the AUV that satisfy the given conditions are possible. That is why recently, increasing attention has been paid to the research and improvement of the AUVs PSC.

For the implementation of many rescue, search or even military operations, the given mission execution accuracy plays a very important role. At the same time, the vehicle stabilized motion accuracy on the given trajectory plays an important role. To achieve maximum accuracy, it is necessary to take into account all the forces affecting the vehicle during motion. To achieve the task and reduce its cost, it is expedient to apply the vehicle motion mathematical modeling, while conducting research on the necessary hydrodynamic characteristics.

At this stage of development of underwater technologies, increasing attention is paid to the “screw–swivel nozzle” propulsion-steering system. This propulsion and steering system has quite effectively proved itself for AUV. However, when changing the vehicle motion trajectory, its hydrodynamic characteristics change. Therefore, it is very important to be able to research them in mathematical modeling.
Latest research and publications analysis. An AUV, operating in trajectory mode, dynamically changes its trajectory, which significantly distinguishes this motion by many characteristics from the rectilinear. An overview of the literature on the AUV trajectory motion indicates the existence of a number of problems solved by the authors in the development of intelligent control systems for various types of underwater vehicles (robots). Having analyzed the most common problems that are currently being solved by AUVs, they note: mapping and review; search operations; study of specified objects and anomalies; underwater position illumination; complex research operations and group operations [1; 2]. It should be noted that the scope of these tasks is constantly expanding [3–6].

At the same time, the authors [7–9] pay attention to the different types of AUV motion trajectories when performing various missions. The following typical modes of the AUV motion trajectories are distinguished: in a straight line — with stabilization on the course, depth, height above the ground; tacks — triangular, rectangular; on a flat curve — on a spiral, on a piecewise broken trajectory, motion along a curve with distance control; height stabilization — along the horizontal structures; displacement stabilization — along the vertical structures, etc. [10].

The parameters determining the operating conditions of the AUV PSC are influenced by the additional hydrodynamic forces and moments that appear on the vehicle hull during the AUV maneuver motion [10–12]. Many authors are drawing attention to the need for a more detailed research of changes in the PSC hydrodynamic characteristics [13–15].

The author [16] noted that the optimal diameter of the propeller nozzle is about 10 % less than the opened one, which reduces the weight and cost of the propeller. The nozzle reduces the probability of the screw being exposed to disturbances and its damages.

Having analyzed the development tendencies of modern steering complexes, it can be concluded that “propeller–rotary nozzle” complex has a number of advantages over other types of PSC, for example, such as “rudder screw” [17; 18].

The “propeller–rotary nozzle” complex has a number of specific features associated with the nozzle effect. Circulation of velocities occurring around the nozzle profile accelerates the development of the propeller flow axial velocity and reduces the load on the propeller, by translating the center part of the complex onto the nozzle. When the complex is operating in the oblique flow, the nozzle forms a velocity field in front of the propeller, aligns it and makes it practically coaxial, maintaining the magnitude of the leakage rate [19].

The analysis carried out shows that today, the relevant question is taking into account the nonlinear peculiarities of the PSC operation in the oblique incident flow conditions, since this mode can be considered the AUV main maneuver operation mode [20; 21].

At present, mathematical modeling is an effective tool for the research of marine moving objects and provides the opportunity to obtain the necessary data without significant costs. This research method is used for modeling the operation of the rotary column of a marine drilling vessel, for the research of moment (torque) and thrust in the development of PSCs, for the research of the self-propelled underwater vehicle spatial motion [22], during the development and optimization of AUV control systems parameters [23; 24]. The accuracy of the result, in this case, will depend entirely on the completeness of the accountable parameters of the mathematical model [25–27].

One of the most commonly used mathematical modeling tools is the MATLAB application package. In this system, there are sufficiently simple tools for solving object-oriented programming tasks [28]. A set of MATLAB and Simulink products allows the creation a model for virtually any system. It is due to these advantages that many authors use the MATLAB application packages [29–31].

The paper [32] presents the mathematical model of the AUV PSC, which allows the research the dynamics of the rotary nozzle operation, but in the terms of processes linearization. LV Kiselev, in his work on the study on the dynamics of the work of driving complexes, also proposes to use mathematical modeling with the construction of the vehicle three-dimensional visualized model based on the design scheme of the Solid Work project, and the use of Symbol Toolbox Matlab for the construction of transient processes in AUV spatial motion.

Separation of previously unsolved parts of the general problem. The above analysis of scientific publications indicates that the main researches of the AUV dynamics are related to their rectilinear motion, or using the linearization of controlled parameters. The question of the AUV curvilinear motion dynamics, which is caused by the rotary nozzle operation in an oblique flow, is insufficiently covered in modern literature. The vast majority of articles are devoted to the research of the propeller column dynamics as an additional control propeller for vessels or marine platforms. Separate publications examine the work of the AUV PSC with the propeller in a fixed nozzle, which only causes rectilinear motion. Some foreign articles describe similar researches, but in the presence of their laboratory equipment. Therefore, the research of the propeller hydrodynamic characteristics in the guide nozzle with the AUV curvilinear motion mathematical modeling, taking into account the processes nonlinearity, is of the current relevance.

THE ARTICLE AIM is to develop a mathematical model of the propeller operation in the guide nozzle in the direct and oblique flow, and make research, in the Simulink system, on the changes in its power characteristics when the AUV is moving in the horizontal plane.

Methods, object and subject of research.

In this work, it is proposed to investigate the AUV curvilinear motion dynamics, which is due to the rotary
nozzle operation in the oblique flow due to the improvement of the underwater vehicle rectilinear motion mathematical model.

Mathematical modeling is an important tool for the research of the changes in the PSC power characteristics in course of AUV maneuvering and for the of automatic control systems elements synthesis, which involves the use of the AUV mathematical models as a control object [33].

The mathematical model of rectilinear motion along the x-axis is presented below. AUV moves under the action of the propeller thrust through a gearbox driven by the main electric propulsion motor. Controlled encapsulated DC (direct current) electric motors with parallel excitation are commonly used as propeller motors on the AUV.

The mathematical model of the control object has the form [20]:

\[
L \frac{di}{dt} = K_u u - r_i i - c\Phi \frac{k_p}{k}\omega;
\]

\[
J_p \frac{d\omega}{dt} = M_{EM} - Q = c\Phi i - K_p \rho D^5 \left(\frac{2\pi}{\omega}\right)^2;\]

\[
(m_{AUV} + \lambda\omega) \frac{dv_x}{dt} = T - F_x - F_{dx} =
\]

\[
= K_T \rho D^5 \left(\frac{2\pi}{\omega}\right)^2 \omega^2 - \frac{cC_s\Omega}{2} v_x^2 - F_{dx};
\]

\[
\frac{dx}{dt} = V_x,
\]

where \(i\) is the instantaneous value of the main propulsion motor armature current, \(u_{i}\), \(K_u\) are the voltage regulator control signal and its gain factor, respectively; \(L, r_i, c, \Phi\) are the electromagnetic parameters of the main propulsion motor; \(\omega\) represents the propeller angular speed; \(k_p\) is the gear ratio \(J_p\) — the moment of inertia of the “propeller–electric motor–gearbox–shafting–propeller screw” system applied to the screw; \(M_{EM} = c\Phi i/k\), is the driving moment of the propeller (main propulsion) motor; \(Q\) represents the braking torque generated by the AUV PSC propeller; \(K_s\) is the non-dimensional non-linear coefficient characterizing the torque of the propeller; \(\rho\) is the water specific density; \(D\) represents the propeller’s diameter; \(m_{AUV}, \lambda\), are the AUV mass and the water added mass; \(V_x\) is the AUV current motion speed along the x axis; \(T\) is the driving propeller’s thrust; \(K_T\) is the dimensionless non-linear coefficient characterizing the propeller’s thrust; \(F_x\) is the water resistance force to AUV movement; \(C_s\) is the AUV hull hydrodynamic coefficient along the x axis; \(\Omega\) — the area of the wetted surface of the AUV outer hull; \(F_{dx}\) is the external disturbance force acting on the AUV hull as it moves along the x axis.

The object of the research is the “propeller–rotary nozzle” in the composition of the AUV PSC. Such a complex is installed on the rotary column, which ensures the rotation of the complex in an arbitrary position relative to the vehicle position, and provides switching at a given angle. The design of the “propeller–rotary nozzle” complex is carried out on the basis of simultaneously satisfying the requirements of mobility and controllability of the vehicle.

The subject of the research includes the hydrodynamic parameters of the PSC, which operates in the downwash water flow during the AUV curvilinear motion. The parameters under research include the propulsive (thrust) force of the mushroom screw and the torque on the shaft.

The propeller thrust force \(T\) along the shaft axis, and also the torque \(Q\) on the shaft, are determined by the formulas:

\[
T = \rho D^4 \left(\frac{\omega}{2\pi}\right)^2 \times
\]

\[
\int_{\frac{r_p}{R}}^{\frac{r_p}{R}} \frac{Z}{8} C_s \left(\frac{b}{D}\right) \left(\frac{2\pi r}{D\omega}\right) ^2 \sin\beta_i (1 - \varepsilon\tan\beta_i) d \left(\frac{r}{R}\right);
\]

\[
Q = \rho D^4 \left(\frac{\omega}{2\pi}\right)^2 \times
\]

\[
\int_{\frac{r_p}{R}}^{\frac{r_p}{R}} \frac{Z}{8} C_s \left(\frac{b}{D}\right) \left(\frac{2\pi r}{D\omega}\right) ^2 \sin\beta_i (1 - \varepsilon\tan\beta_i) d \left(\frac{r}{R}\right).
\]

The integrals in the above formulas are called the propeller thrust coefficient \(K_s\) and the screw torque coefficient \(K_t\), respectively. These coefficients are presented in the form of the propeller action curves and characterize the thrust and moment in various screw operation modes: \(T = K_s \rho D^4 \left(\frac{\omega}{2\pi}\right)^2\); \(Q = K_t \rho D^4 \left(\frac{\omega}{2\pi}\right)^2\); where \(\rho\) is the specific density of water; \(\omega\) — angular velocity of rotation; \(D\) is the screw diameter.

Basic material.

Mathematical model of “propeller–rotary nozzle” operation

When constructing AUVs, the propellers (PS) in the nozzles are used for propulsion and control means. Such a complex is installed on the rotary column, which provides the rotation of the complex relative to the vehicle position, and provides a turning angle of 0 degrees to 360 [13].

As a result of the change in the propeller incidence flow angle, the hydrodynamic characteristics of the PSC vary considerably. Thus, when the nozzle is shifted to an angle \(\delta\), the symmetry of its flow around the motor is broken, which leads to the appearance of the flow velocity \(\vec{U}_n\) component, the average direction of which can be taken perpendicular to the nozzle axis. Then, the complex flow axis, according to equality \(\vec{U}_n = \vec{U} - \vec{U}_n\), deviates from the propeller shaft axis in the same direction as the nozzle at a certain angle \(\delta\). This angle depends on the nozzle relative length \(\overline{L}_n\) and can be expressed as follows:

\[
\delta = \theta_n \delta_n = (1 - a_n \delta_n) \delta_n,
\]
where \( a_n \) is the coefficient of approximation, which for nozzles without a stabilizer, is determined by the expression:

\[
a_n = 0.04838 - 0.067865 \overline{L}_n + 0.023328 \overline{L}_n^2.
\]

The design of the “screw–rotary nozzle” complex is based on the simultaneous satisfaction of the requirements of mobility and controllability of the vehicle. The diameter of the PS, the coefficients of the expansion of the nozzle are determined from the calculation of mobility, and the relative length and size of the nozzle — from the requirements of controllability. If the optimal relative length of the running nozzle is 0.55–0.65, then the relative length of the rotating nozzle is increased to 0.8–1.0 [12]. Structurally, the complex is shown in fig. 1.

The relative length of the nozzle is determined as follows:

\[
\overline{L}_n = \frac{L_n}{D_n}.
\]

At the transfer of the nozzle (Fig. 2), the complex thrust \( \overline{P}_R \) is divided into two components: the reaction deviated at the angle \( \delta_r \) of the flow \( \overline{P}_s \), and the additional reaction \( \overline{R}_R \) that occurs when the part of the flow runs over the deflected profile of the rotary nozzle. Therefore:

\[
\overline{P}_R = \overline{P}_s - \overline{R}_R. \quad (1)
\]

Designing equality (1) on the diametrical and horizontal plane (see fig. 2) allows to obtain the following result [13]:

\[
P_x = P_s \cos \delta_r = P_s \left(1 - \sin^2 \left(\theta_n \delta_n \right)\right); \quad (2)
\]

\[
P_y = P_s \sin \delta_r = P_s \sin \left(\theta_n \delta_n \right) \cos \left(\theta_n \delta_n \right). \quad (3)
\]

Then

\[
P_s = \sqrt{P_x^2 + P_y^2}. \quad (4)
\]

Due to the increase in the stop force, as the angle of flow changes, so does the effort on the propeller shaft.

In this case, the torque on the propeller shaft can be defined as follows [14]:

\[
M'_M = M'_c \left(1 + \sin^2 \left(\theta_n \delta_n \right)\right), \quad (5)
\]

where \( M'_c \) is the torque of the screw in the coaxial flow.

Thus, by adding the mathematical model of the AUV rectilinear motion [7] with the formulas presented above, the model of the AUV horizontal motion is obtained with the propeller in the nozzle for flat circulation.

**General characteristics of the developed model**

The control object in the horizontal motion mode is considered as a solid body (AUV hull), which moves under the thrust action of the horizontal propulsion, driven by a propeller DC electric motors with independent excitation. Control is performed by applying to the motor armature an appropriate supply voltage \( U = K_u u \), where

\( K_u \) is the voltage source gain factor, \( u \) is the required voltage value. A mathematical model of the control object is presented in [7; 15].

To simulate the AUV motion and research the systems being created, the Simulink environment of the MATLAB 2010a environment is used. On the basis of AUV mathematical models and PSC of “propeller-rotary nozzle” type, a Simulink-model was created that simulates the AUV movement in a horizontal plane on a flat circulation.

The general view of the model is presented in Fig. 3.

To implement the nozzle rotation at any installation angle to the model a “sigma” block was added to the model (Fig. 4). This block simulates the smooth rotation of the nozzle at the operator-specified angle and keeps it at a given position. The connection of this block provides the possibility of simulating the AUV movement on a flat circulation in the horizontal plane.

Figure 5 shows the structure of the main “AUV model” block. The “Equation1”, “Equation2”, and “Equation3” blocks, in the form of recurrent equations, implement the AUV motion mathematical model in the horizontal plane, PSC which consists of a propeller operating in a coaxial flow. A detailed description of the operation of these blocks is presented in [7], [15]. The structure of the “Subsystem1” block is presented in fig. 6.

The “Subsystem1” block is conditionally divided into two parts (see Fig. 6). In the first part, calculations are made of the propeller thrust force value \( T \) along the shaft axis and the torque \( Q \) on the shaft according to the following formulas [7]:

\[
T = K_T \rho D_s^4 \frac{\omega^2}{4\pi^2}; \quad Q = K_Q \rho D_s^5 \frac{\omega^2}{4\pi^2},
\]

where \( D_s \) is the inner diameter of the nozzle; \( D_s \) is the screw diameter; \( \overline{L}_n \) is the nozzle length.
Fig. 3. General view of the AUV model

Fig. 4. Structure of “sigma” block

Fig. 5. The structure of “AUV model” main block

Fig. 6. The structure of the “Subsystem1” block
where $\rho$ is the water specific density; $\omega$ represents the propeller angular speed; $D$ is the diameter of the screw.

The coefficients $K_T, K_Q$ represent the screw thrust coefficient and the screw torque coefficient, respectively. These coefficients are presented in the form of the propeller action curves and characterize the thrust and torque when the screw operates in a coaxial flow [7].

The second part of the block is designed to simulate the PSC “propeller–rotary nozzle” operation, and implements the equations (2)–(5) presented above. The functions $Fcn, Fcn1, Fcn2, Fcn3$ in the Simulink system are implemented by the following formulas:

$Fcn$: \[ T_c = u(1) \cdot (1 - \sin(u(2) \cdot 0.6) \cdot (2)) \],

$Fcn1$: \[ T_p = u(1) \cdot \sin(u(2) \cdot 0.6) \cdot \cos(u(2) \cdot 0.6) \],

$Fcn2$: \[ T = \sqrt{u(1)} \cdot (2 + (u(2) \cdot (2))) \],

$Fcn3$: \[ Q = u(2) \cdot (1 + (\sin(u(1) \cdot 0.6) \cdot (2))) \].

At the block input, data are received about the current propeller rotational speed values and the vehicle speed. At the output, we have the listed values of the propeller thrust force $T$ and the torque $Q$ on the shaft, which correspond to the propulsion and steering complex operation in the incident flow.

**Results**

To test the performance of the developed mathematical model and its Simulink implementation, we will carry out research on the effect of the oblique flow of water incident on the PSC of the “propeller–rotary nozzle” type, for AUV with the following characteristics:

- mass — 60 kg;
- travel speed — 1 m/s;
- propeller diameter — 160 mm;
- diameter of the rotary nozzle — 166.4 mm;
- the relative length of the rotary nozzle — 1.0.

Figure 7 shows the result of the operation of the sigma block.

In fig. 8 the result of simulation of AUV movement, at a speed of 1 m/s is presented.

**Discussion of the received results.** Consequently, according to the results of the work, it is evident that the developed mathematical model gives an opportunity to study the hydrodynamic parameters of the propulsion

![Fig. 7. Change in the nozzle rotation angle](image1)

![Fig. 8. Simulation results of AUV movement on flat circulation:](image2)

- a — speed diagram;
- b — shaft rotational speed diagram;
- c — diagram of propeller thrust change;
- d — graph of the torque on the shaft
and steering complex, which operates in the downwash flow during the AUV plane curvilinear motion. It is seen from the presented graphs that the change in the nozzle rotation angle significantly affects all the AUV controlled parameters.

In this case, AUV motion simulation on a flat circulation at a speed of 1 m/s was carried out. When the specified value is reached

The graph (see fig. 7) shows that the vehicle begins to move in a straight line, after some time, the nozzle is rotated through an angle of 35°. When the specified value is reached, the vehicle starts circulation with a constant angle. Further, the AUV starts operating in the oblique flow, that is incident to the propeller. Due to the change in the flow angle, the propeller thrust force changes (see fig. 8), the force on the propeller shaft increases, and the vehicle speed decreases. The obtained result indicates that this maneuver will negatively affect the accuracy of the AUV trajectory movement. Therefore, in the control system synthesis of the AUV trajectory motion, it would be desirable to compensate for this effect. Using the presented model, it is possible to obtain a data sample — the dependence of the change of the propeller thrust on the vehicle movement speed and the incident flow angle. This dependence will have a substantially nonlinear character and can be taken into account in the AUV control system synthesis.

In the work of Gladkovoy A.I. and V.Veltishchev, the results of modeling the AUV PSC dynamics obtained in the Simulink MATLAB system are presented. However, these results were obtained in the absence of nonlinear effects. To correct the influence of the nonlinearity characteristics of the mushroom screw, it is proposed to choose the value of the input signal that will bring the output signals to a linear form.

One of the advantages of the work I presented is the consideration of the inconsistency of the propeller operation characteristics, embedded in the AUV mathematical model [20; 22].

CONCLUSIONS. 1. The AUV dynamics model has been improved by supplementing its mathematical model with equations describing the PSC “propeller–rotary nozzle” operation.

2. A mathematical model of functioning AUV propulsion and steering complex of the type “propeller–rotary nozzle” in the incident flow was developed and was simulated in the Simulink system to change the power characteristics on the AUV flat circulation in the horizontal plane.

3. It has been established that the thrust of the AUV PSC of the “propeller–rotary nozzle” type in the oblique incident flow is nonlinear depending on the water flow angle and the AUV speed.

4. The dependences obtained can be taken into account in the synthesis of the automatic control systems regulators for AUV flat maneuvering (shunting) movement.

Список літератури


